1. When the input to a causal LTI systems is

$$x[n] = -\frac{6}{7}\left(\frac{1}{3}\right)^n u[n]$$

the z-transform of the output is

$$Y(z) = \frac{z}{(1-\frac{3}{2}z^{-1})(1-z^{-1})(1+\frac{1}{2}z^{-1})}$$

(a) Find the z-transform of $x[n]$.

(b) What is the region of convergence of $Y(z)$.

(c) Find the impulse response of the system.

(d) Is the system stable? Why or why not?

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a

$$X(z) = -\frac{6}{7} \cdot \frac{1}{1-\frac{1}{3}z^{-1}} + \frac{6}{7}\left(\frac{1}{1-\frac{3}{2}z^{-1}}\right)$$

$$= \frac{-\frac{6}{7}(1-\frac{1}{2}z^{-1}) + \frac{6}{7}(1-\frac{1}{3}z^{-1})}{(1-\frac{1}{3}z^{-1})(1-\frac{2}{3}z^{-1})} = \frac{-\frac{6}{7}+\frac{6}{7}+\frac{18}{14}z^{-1}-\frac{6}{21}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-\frac{2}{3}z^{-1})}$$

$$= \frac{\frac{54}{42}z^{-1}-\frac{12}{42}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-\frac{2}{3}z^{-1})} = \boxed{\frac{z^{-1}}{(1-\frac{1}{3}z^{-1})(1-\frac{2}{3}z^{-1})}} = X(z)$$

b

The system is causal, so $Y(z)$ must be causal as its output, $\therefore$ $Y(z)$ is right-sided

So $\boxed{ROC: \ |z| > \frac{3}{2}}$